MATH 1105 - FALL 2008 - 10-24-08
SECTION 2, QUIZ 2

You have 25 minutes to complete the following problems. No notes or calculators are allowed or necessary. This quiz is out of 35 points. Good Luck!

IMPORTANT: Do NOT simplify your answers, at all. Leave answers in choose/permutes notation.

(1) (10 Points)
   a. How many possible arrangements are there of the letters in the word "letter" assuming that you cannot tell letters of the same kind apart?
   $\frac{6!}{2!2!1!1!}$
   b. An arrangement of letters is clumped if the arrangement is made up of blocks of letters of single types. For example, "ttelrr" is clumped and "rerttl" is not. How many possible clumped arrangements are there for the letters in the word "letter" assuming that you cannot tell letters of the same kind apart?
   4!
   c. Given that an arrangement is clumped what is the probability that the arrangement begins with the letter "e"?
   $\frac{3!}{4!}$

(2) (10 Points) The class 2012 at Miniscule College has 20 students. In each of the 4 years at Miniscule College, the students in the class of 2012 will be assigned an order at random in which to select their dorm rooms. We assume that each ordering is independent of the orderings in other years, no students drop out of Miniscule College, and everyone lives on campus.
   a. How many different orderings are there?
   20!
   b. Jill, Tom, and Cynthia are students in the Class of 2012 at Miniscule College. What is the probability that Jill, Tom and Cynthia will be the first three students in the order? (Hint: Jill does not have to be first)
   $\frac{3!17!}{20!}$
   c. What is the probability that Jill, Tom, and Cynthia will be the first three students in the order for exactly three of the four years?
   $\binom{4}{3}(\frac{3!17!}{20!})^3(1-\frac{3!17!}{20!})$

(3) (15 Points) There are 4 seniors in a group of 10 students. From this group of 10 a committee of 5 students is chosen, though at least 3 of the 5 students on the committee must be seniors.

a. How many different possible committees are there? (Hint: Total number of possible committees = Total number of possible committees where exactly 3 seniors are on the committee + Total number of possible committees where exactly 4 seniors are on the committee.)
$\binom{4}{3}\binom{6}{2} + \binom{4}{4}\binom{6}{1}$

b. Ben is one of the 4 seniors in the set of 10 students. How many different possible committees are there with exactly 3 seniors on the committee and Ben is one of the?
$\binom{3}{2}\binom{6}{2}$

c. If a possible committee is chosen at random, what is the probability that there are exactly 3 seniors on it and Ben is one of them?
$\frac{\binom{3}{2}\binom{6}{2}}{\binom{4}{3}\binom{6}{2}+\binom{4}{4}\binom{6}{1}}$

d. Given that Ben is on the committee, what is the probability that there are exactly 3 seniors on the committee?
$$\frac{\frac{\binom{3}{2}\binom{6}{2}}{\binom{4}{3}\binom{6}{2}+\binom{4}{4}\binom{6}{1}}}{\frac{\binom{4}{4}\binom{6}{1}}{\binom{4}{3}\binom{6}{2}+\binom{4}{4}\binom{6}{1}}+\frac{\binom{3}{2}\binom{6}{2}}{\binom{4}{3}\binom{6}{2}+\binom{4}{4}\binom{6}{1}}}$$

Bonus. (3 Points) Given 5 cards chosen at random from a deck of 52 cards, what is the probability that the 5 cards constitute a "high card" poker hand? More specifically, a hand is a "high card" hand if it does not contain any of the following: a pair, two pairs, a three of a kind, a full house, a straight, a flush, a four of a kind, or a straight flush.
$(\binom{13}{5} - 10)(4^5 - 4)$